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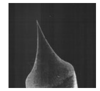
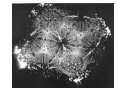


High-Electric-Field
Quantum Science
& Technologies



UNIVERSITY OF SURREY

UPDATE ON FIELD ELECTRON EMISSION SCIENCE AND TECHNOLOGY 2026

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Prologue

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[For a list, websearch title: Support for Murphy-Good FE theory in textbooks, reviews, etc.]

Structure of talk

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There are at least 28 textbooks and review articles relating to the topic.
[For a list, websearch title: Support for Murphy-Good FE theory in textbooks, reviews, etc.]

I hope to provide a high-level overview of a small part of this material.

Overall structure of talk is:

Part A: Introduction: terminology & basic ideas

Part B: Technological and other applications

Part C: Field electron emission science

Part D: Problems with FE literature

Part E: Some future needs

Nomenclature issues: fields

The emission process of interest has three alternative names:

field emission, or
field electron emission, or
electron field emission.

Due to relationships with field ion emission, I prefer field electron emission.
But I use "FE" as a common abbreviation for all three alternatives.

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Lasers are increasingly used in some applications involving field emitters. To make a clear distinction between the electric component of a travelling electromagnetic (EM) wave and the static fields used in FE, I now refer to these static fields as an electrostatic (ES) field (rather than an "electric field").

[The theories of electron emission induced by EM and ES fields appear to be different, at least to some extent.]

Sign convention issues

In conventional classical electrostatics (with distance increasing positively to the right), all of ES fields, currents and current densities in FE are negative in sign.

The electron emission sign convention is almost universally used in FE literature and is used here. In this convention, ES fields, currents and current densities in FE are taken as positive.

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Following Fowler & Nordheim, I use the symbol *F* to denote a quantity that is the negative of classical ES field.

Strictly, this symbol *F* represents the electrostatic potential gradient (ESPG). Here, I follow the usual convention of simply referring to *F* as the "(ES) field".

Local emission current density (LECD) J_L

A parameter of major interest is the local emission current density (LECD) J_L, at some surface location "L" where the emitting surface has:
local work function ϕ ; local surface ES field (ESPG) F ;
and thermodynamic temperature T .

There are many different theories and equations for J_L, corresponding to different physical assumptions and different mathematical approximations. In all cases, the formulae can be understood in the following way:

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Emitted electron behaviour

[depends on ϕ, F, T, for planar emitters]

=

Electron statistical mechanics inside emitter

[depends on temperature]

×

Barrier transmission probability at emitter surface

[depends on barrier shape and electron normal-energy]

Murphy-Good theory as best working theory

At present, for many technological applications, the most useful *working theory* is Murphy-Good (MG) low-temperature FE theory, initially introduced in 1956. This gives the LECD as

$$J_L \approx \lambda_T \, t_F^{-2} \, a\phi^{-1} F^2 \exp[-v_F b \phi^{3/2} / F] ,$$

where *a* and *b* are the Fowler-Nordheim universal constants, v_F and t_F are mathematical factors that are particular values of well established mathematical functions, and λ_T is a temperature correction factor. Both λ_T and t_F^{-2} have values relatively close to unity and one or both may sometimes be omitted.

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MG FE theory is itself an approximation that is urgent need of being replaced by better working theory, in particular by packages (such as "GETELEC") that aim to solve the Schrödinger equation numerically. However, as of now (2026), MG FE theory is probably still "good enough for many technological purposes".

"Murphy-Good", NOT "Fowler-Nordheim"

The 1956 Murphy-Good low-temperature FE equation has often been called the "Fowler-Nordheim equation", particularly in some older literature.

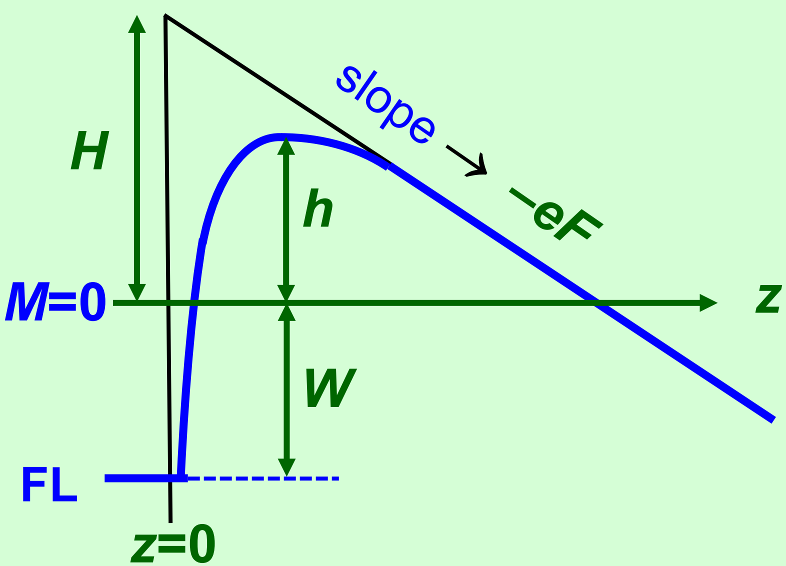
This practice is misleading because MG FE theory predicts much higher current densities than the original 1920s FE theories (by a factor of 100 or more).

Use of the name "Fowler-Nordheim equation" for the 1956 Murphy-Good low-temperature equation now needs to be discouraged.

The Schottky-Nordheim (SN) barrier

Schottky-Nordheim (SN) barrier

[vertical axis shows potential energy and electron normal-energy]



$$M(z) = H - eFz - 1/16\pi\varepsilon_0 z \quad (z>0)$$

Murphy-Good FE theory uses the Schottky-Nordheim (SN) barrier, shown alongside.

However, if the electron normal-energy *W* (relative to the Fermi-level FL) is large and the Schottky-barrier height *h* is small, then the transmission theory used in MG low-temperature FE theory breaks down.

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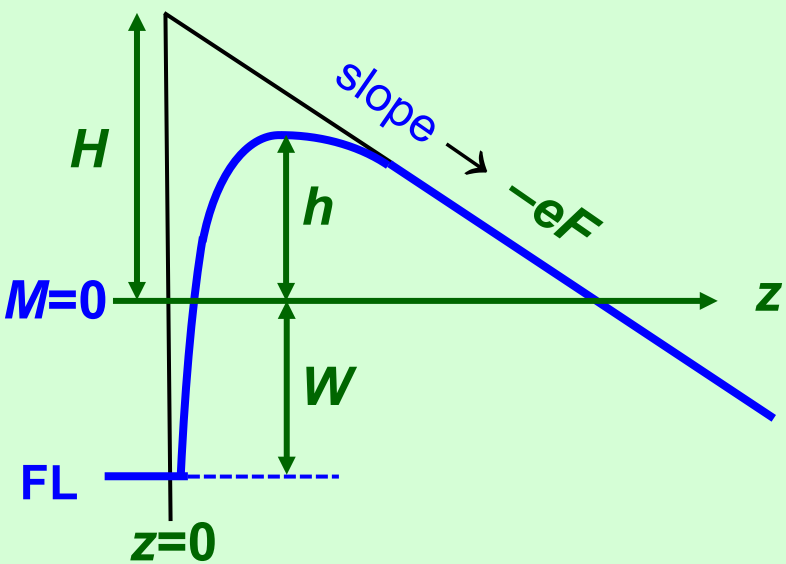

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Hence MG low-T FE theory breaks down at sufficiently high temperatures, when many electrons tunnel close to the top of the barrier.

Hence, MG low-T theory has a regime of validity.

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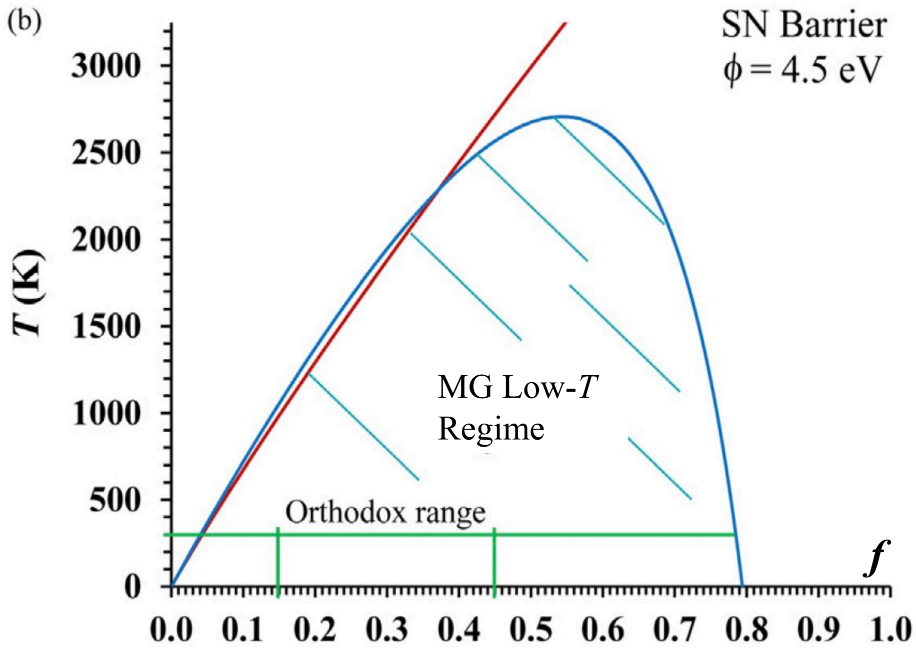

The diagram alongside, provided by Dr M.M. Allaham (Brno), shows the conventionally predicted regime of validity for the MG low-*T* equation, for a 4.5 eV emitter.

[The scaled field $f \equiv F/F_R$, where F_R is the field needed to pull the top of the SN barrier down to the Fermi level.]

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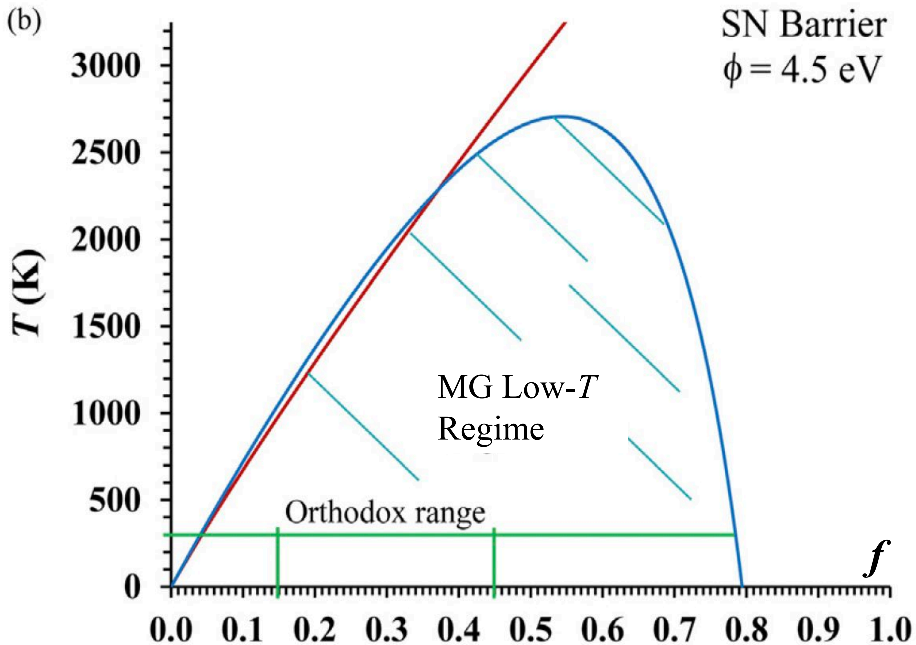

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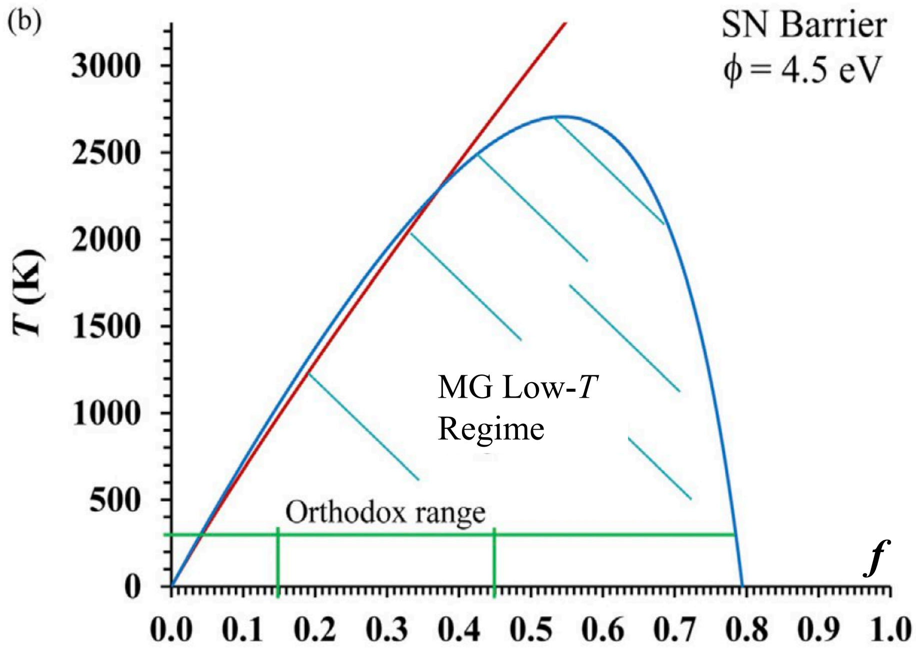

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This regime has been called by several different names including: field emission (FE); cold field emission (CFE); thermal-field emission; Fowler-Nordheim FE: low-temperature FE; deep tunnelling regime.
Today's talk relates to emission in this regime.

B

Technological and other applications

Relevances of field electron emission

The main relevances of field electron emission (FE) are:

1. **FE provides underlying theory for many technologies that are: either (a) in use; or (b) speculative; or (c) now failed.**
2. **FE is part of the theory of electrical breakdown in vacuum.**
3. **FE is one of the paradigm examples of quantum tunnelling.**

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1. For FE-based technologies, related theory can be used to:

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(d) Interpret experimental data.

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2. In electrical-breakdown contexts, main uses are to predict emission currents and resultant emitter heating, often as part of discussion of proposed breakdown mechanisms.

[But more-general theories of "quasi-electrostatic FE" (FE induced by oscillating ES fields) and of thermal-field-emission effects can be more useful.]

Relevances of field electron emission

3. In the context of quantum tunnelling:

(a) Some aspects of FE theory suggest that the presentation of quantum mechanics could be improved by the statement of "preliminary assumptions".

(b) A few aspects of FE currently have no good explanation in terms of existing quantum mechanics
[e.g., see: https://doi.org/10.48550/arXiv.2505.00872]

(c) Any fundamental new developments in FE theory might have wider implications. [For example, DNA mutation involves quantum tunnelling of protons.]

Because of its various contexts, continued development of FE theory is a matter of scientific interest.

Electron sources & technologies: overview

The two main types of FE electron source are:

(a) single tip field emitters (STFEs): a single needle-like or post-like emitter;

(b) large area field electron emitters (LAFEs): a large-area electron source containing many or very many individual emission sites; this can take various different forms (see later).

[Cylindrical wire emitters have been used in the distant past.]

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The two main geometrical configurations (particularly with STFEs) are:

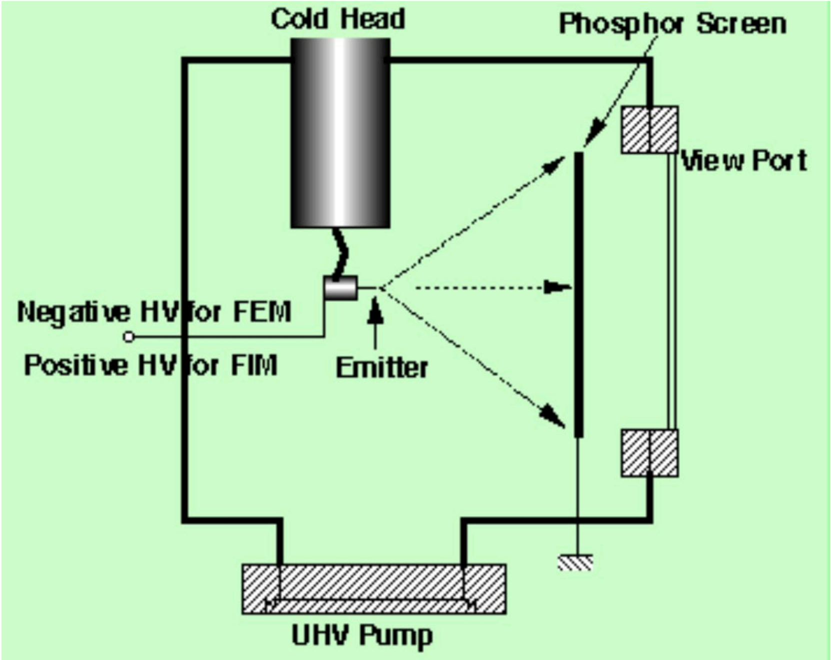
(a) electron source distant from counter-electrode ("large-gap geometry");

(b) electron source close to counter-electrode ("small-gap geometry")

The electrostatics of these two configurations is significantly different, and tidying it up is an active research topic.

Historical technologies: FEM & FIM

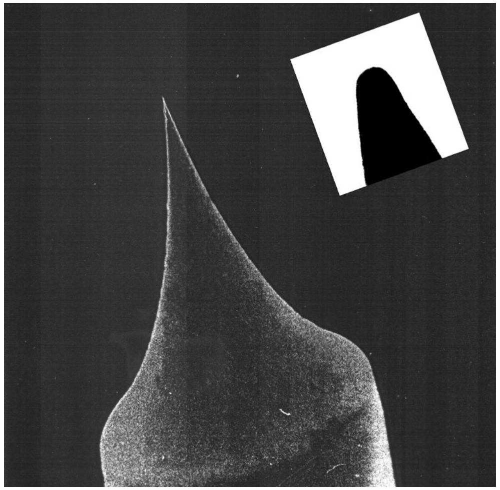
The "historical technologies" of field electron (emission) microscopy (FEM) (1930s) and field ion microscopy (FIM) (1950s) both used a needle-type emitter facing a phosphor screen. Both used similar basic apparatus, shown (very) schematically below, but cooling is only needed for FIM.



For FIM, a low-pressure operating gas (typically He) needs to be present.

[diagram courtesy Wikipedia]

Field electron and ion microscopy



Needle-shaped field electron/ion emitter

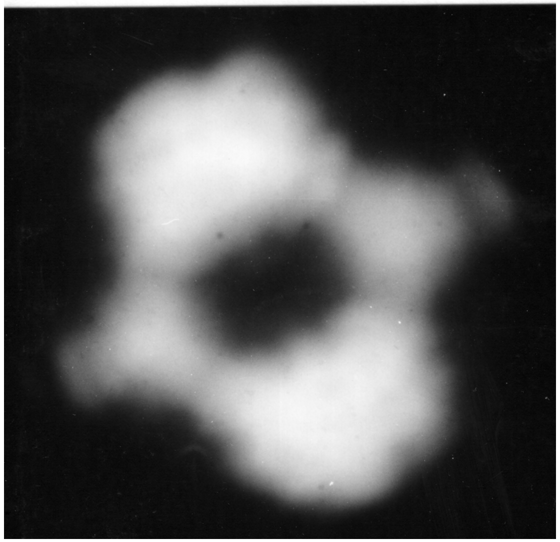
[Imaged by electron microscopy.]

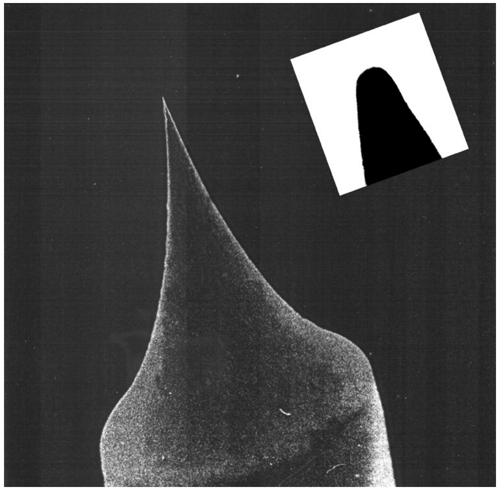
Field electron and ion microscopy

Electron emission gives field electron micrograph (110-oriented tungsten)

[Bright regions show high field or low work function.]

Needs –ve surface ES fields of order 2 to 4 V/nm.





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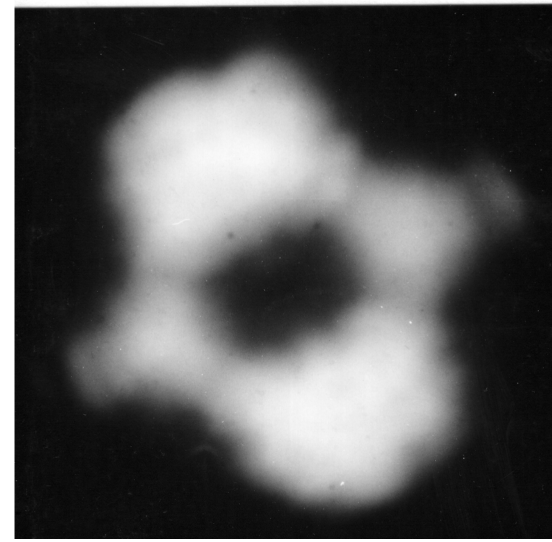
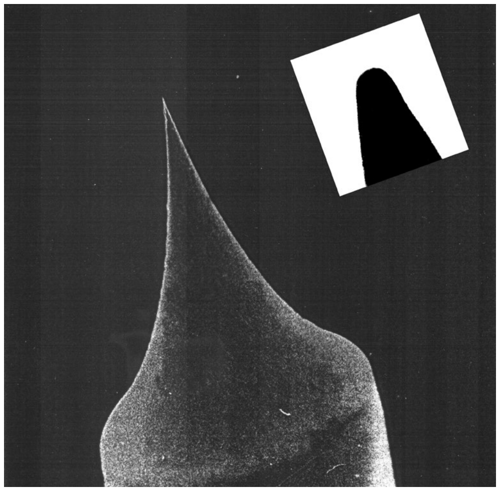




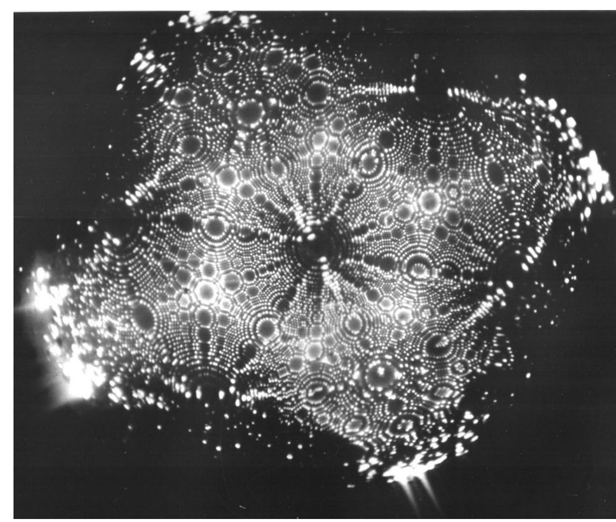
Needle-shaped field electron/ion emitter

[Imaged by electron microscopy.]

Ion emission gives
field ion micrograph.
(He on W image)

[Bright spots show individual surface atoms.]

Needs +ve surface ES fields of about 45 V/nm.



Field electron and ion microscopy

FIM and FEM are now obsolete as working research techniques, but can still be useful for special purposes – in particular assessing the structure of the emitting apex of a field electron or field ion source.

Emitter materials & shapes

In all cases, a key requirement is to get enough electrostatic (ES) field enhancement, so that the operating voltage is "not too high" .

Needle/Post geometry

- **Metal needles**
- **Carbon nanowhiskers**
- **Other non-metals (e.g., ZnO, HfC)**
  - **[formed as nanowires and then etched]**

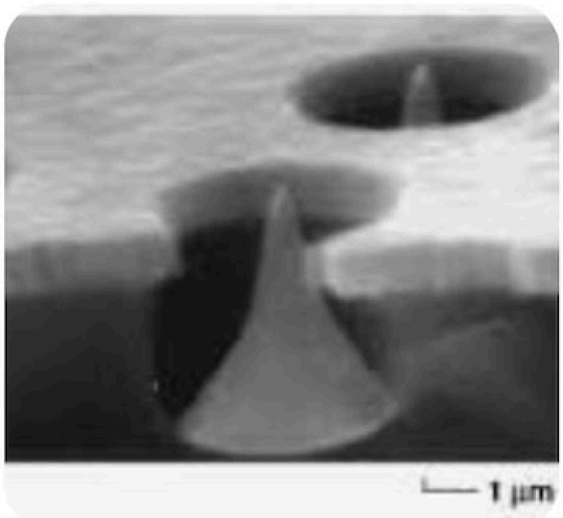
Large-area geometries (most prominent historical & recent examples)

- **Spindt arrays**
- **Carbon nanotubes (CNTs) in holes**
- **Post arrays**
- **Carbon mats**
- **Carbon fibres and yarns**
- **Graphene-edge devices**

Emitter materials & shapes

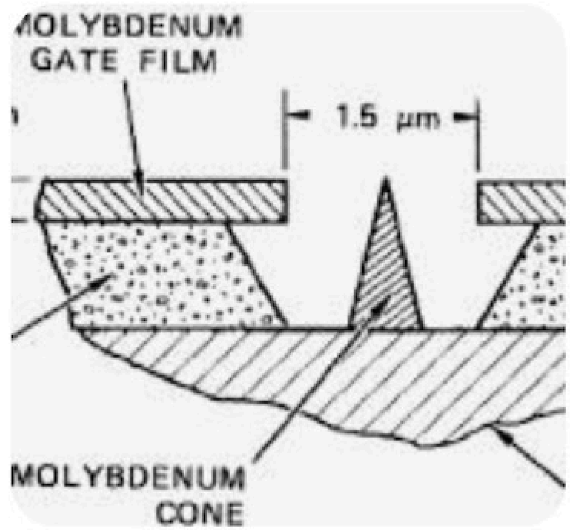
Here are a few example\#.

A Spindt array



File:Fig218 Field Emitter Array....

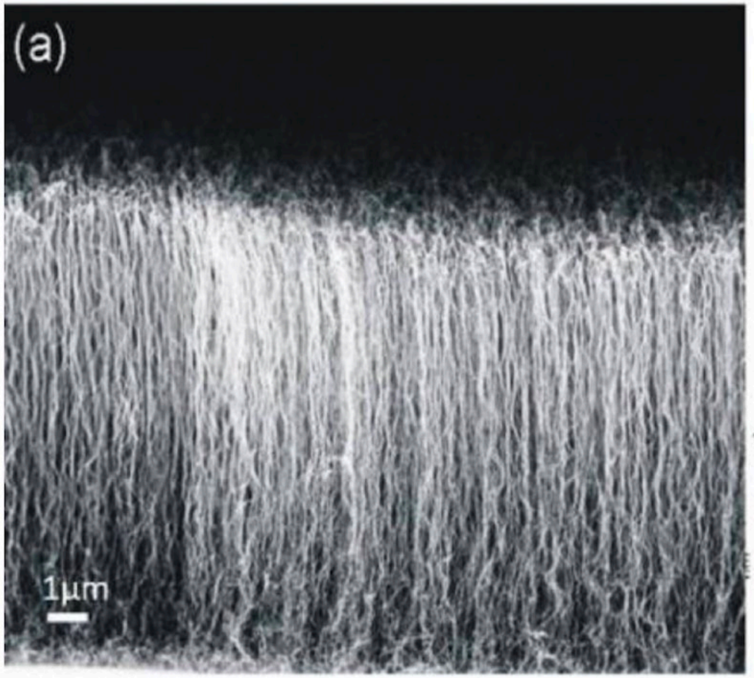
Wikipedia



Spindt Cathodes and Other Fi...

Springer Nature

One form of "Carbon Mat"



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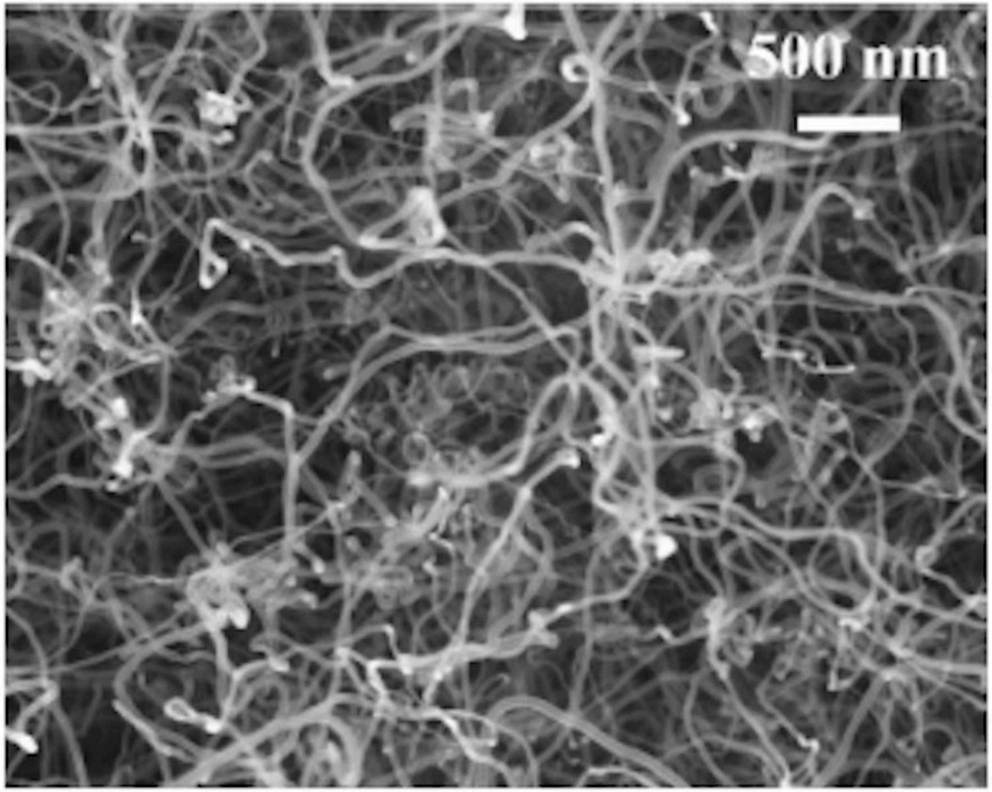
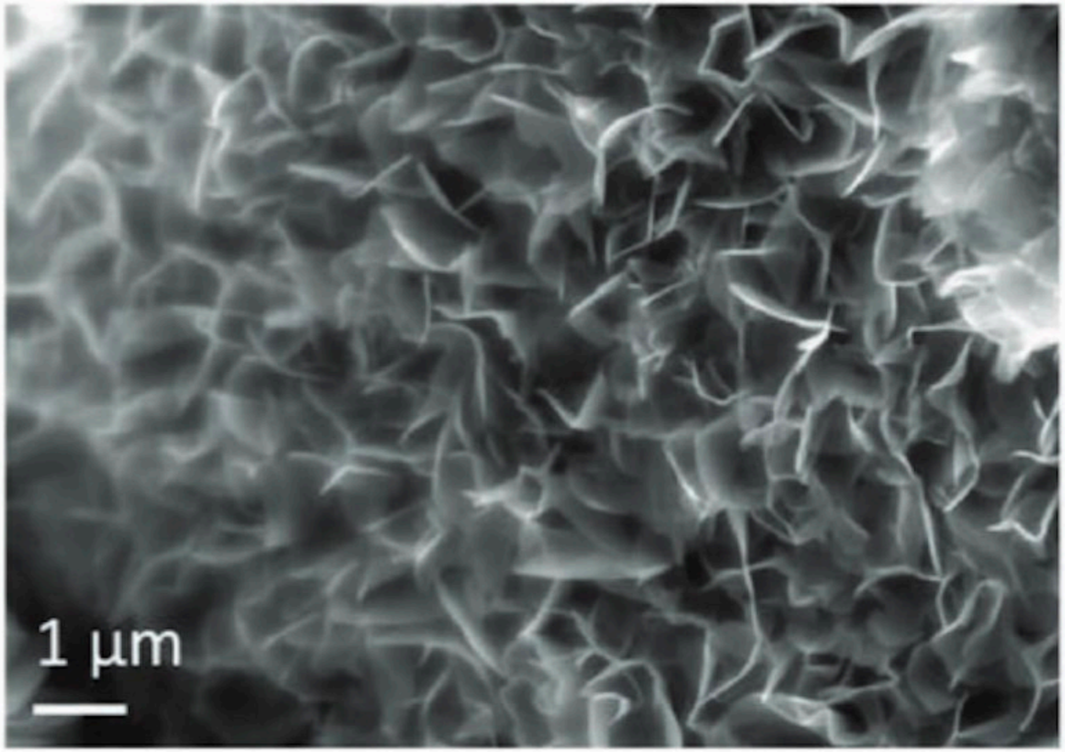
Another carbon structure



Structure based on graphene edges



Applications - single-needle/post emitters

Single-needle/post emitters – large gap

- **Electron microscopes**
- **Other electron beam-instruments – e.g. electron-beam lithography**
- **Theoretical model in electrostatics calculations**
- **Theoretical model in vacuum (electrical) breakdown**
- **Theoretical model for lightning rod and corona discharges**

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Single-needle emitter – small gap

- Topographiner [did not work well enough] (R.D. Young)
- Near field emission scanning electron microscopy (D. Pescia)
- Low-energy point-source microscope (H.W. Fink)
- Scanning probe lithography

[Obviously, this group of applications is closely related to scanning tunnelling microscopy, but the emission theory is slightly different.]

Applications – large-area sources

Large-area-source applications (RGF's assessment)

(a) Established

- **Some types of computer flash memory**
- **X-ray generators for medical applications**
- **Ionization devices for miniature mass spectrometers**
- **Theoretical models for electrostatic simulations**

(b) Promising

- **X-rays generators for other tasks**
- **Sensors**
- **Radiation-hard air-channel or vacuum-channel transistors**

(c) Maybe

- **Microwave generators and similar devices**
- **Spacecraft neutralization applications**
  (especially in connection with electric propulsion)
- **Tether-type spacecraft propulsion devices**

cont./

Applications – large-area sources

Large-area-source applications (RGF's present assessment) (cont.)

(d) Unlikely to succeed

- **Field emission lamps**
- **Electrostatic-field-energy storage devices**

(e) Apparently failed technology

- **Field emission displays**

Failure mechanisms

Field electron emitters can be tricky to use because they can degrade or fail, particularly when operated:

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Adsorbates on the emitter surface can, in particular cases:

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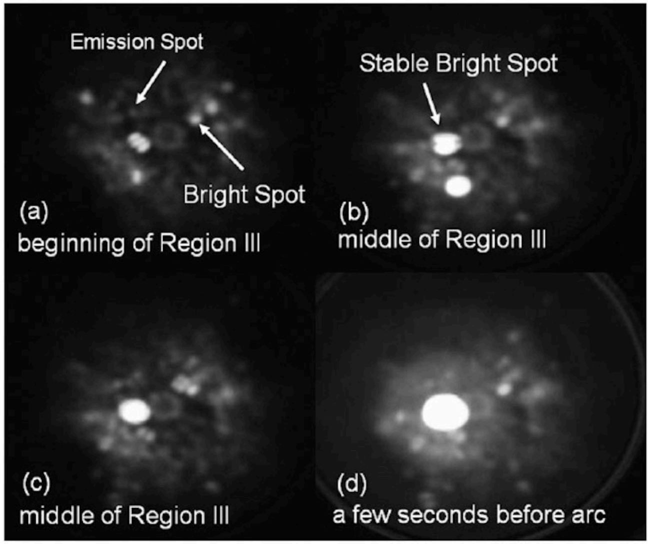
The large bright spot shows a carbon-based nano-whisker growing on top of a pointed-needle electron-microscope source.

These whiskers may have to be "burnt off" at regular intervals (e.g., overnight).

[Diagram courtesy: K.S. Yeong & J.T.L. Thong, J. Appl. Phys. 99 (2006) 104903, fig. 8.



Failure mechanisms

Excessive current and resulting excessive emitter heating can cause:

- **emitter shape change (emitter get more pointy) as a result of "electroformation" ("thermal-field shaping"), eventually leading to emitter explosion or other breakdown mechanisms;**
- **with carbon-post emitters, "chew-off" effects, assumed to be caused by Maxwell-type stresses.**

Self-healing

A useful property of an emitter is if it can "self-heal".

Thus, carbon-yarn emitters can "split apart" and create new "emitting tendrils".

C

FE electron emission science

FE electron emission science

Field electron emission science

I use the name "field electron emission (FE) science" to cover the totality of all "theory" specifically relevant to FE-related situations and apparatus

At a top level, I divide FE science into

1. **Emission physics**
2. **Field emitter electrostatics**
3. **FE systems engineering.**
4. **Data analysis issues**

The elementary FE equation

As stated earlier: at present, for many technological applications, the most useful *working theory* is Murphy-Good (MG) low-temperature FE theory. This gives the LECD as

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There is an older theory, which I call the elementary (EL) FE equation. This theory is a simplified version of 1920s FE theory. It doesn't have the mathematical factors in and takes the form

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As compared with MG low-T theory, elementary FE theory underpredicts local emission current densities by factors typically more than 100.

Unfortunately, this misleading elementary equation is still stated quite often in FE literature. [This practice needs to be discouraged.]

Testing FE theory against experiment

There are good theoretical scientific arguments that Murphy-Good low-*T* FE theory is better physics than (and numerically superior to) elementary FE theory. This view is presented in at least 28 books and review-type articles.

However, it seems that some experimentalists would be more convinced by an experimental argument.

Testing FE theory against experiment

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However, it seems that some experimentalists would be more convinced by an experimental argument.

In the last 100 years there have been tens of thousands of experiments on the electrostatics of field emitters, but (until recently) only ONE more-or-less successful attempt to test current-density FE theory against experiment [by Ehrlich and Plummer in 1978].

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Recently, a second method has been proposed and implemented by Shuai Tang and colleagues, at Sun Yat-Sen University. A relevant paper, entitled "Experimental methods for characterization of atomic-scale field emission point-electron source" is shortly to be published in Phys. Rev. Appl.

Both experimental methods tend to:

(a) confirm that MG low-*T* FE theory is superior to elementary FE theory;

(b) suggest that MG low-*T* FE theory overpredicts experimental current densities [– probably because "there are no atoms in MG theory"].

The limited validity of MG low-T FE theory

The MG low-temperature FE equation in fact has limited validity, for several different reasons:

(a) it is adequately valid only in a limited regime of field and temperature;

(b) Its derivation involves the mathematical approximation of taking only the first term in a Taylor expansion.

These problems can, in principle be solved by using better quantum mechanics and numerical methods, for example the "GETELEC" program.

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Other problems arise because MG theory is a theory of emission from smooth planar surfaces. One needs to consider:

(c) how to develop theory for sharply curved surfaces, and how to specify limits for when smooth planar theory breaks down. [Relevant theory already exists and more is under development: the following talk by Prof. Xanthakis will deal with this issue.]

(d) how to get atomic-level wave-functions and band-structure effects into emission theory. [There is some relevant work, but more attention is needed.]

(e) there appear to be some deeper quantum-mechanical problems in developing "exactly correct" theory.

Some recent electrostatics results

The electrostatics of post-like objects plays an important role in FE theory, particularly in geometries where the post is cylindrically symmetric with a rounded apex and stands on one of a pair of parallel planar plates, and the post height *h* is very much less than the plate separation.

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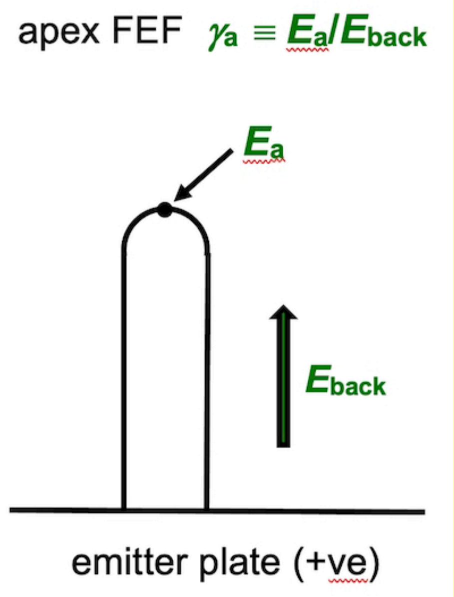
A parameter of particular interest is the apex field enhancement factor (apex FEF) $\gamma_a \equiv E_a / E_{back}$, where E_a is the ES field at the post apex and E_{back} is the background ES field that would exist in the absence of the post.



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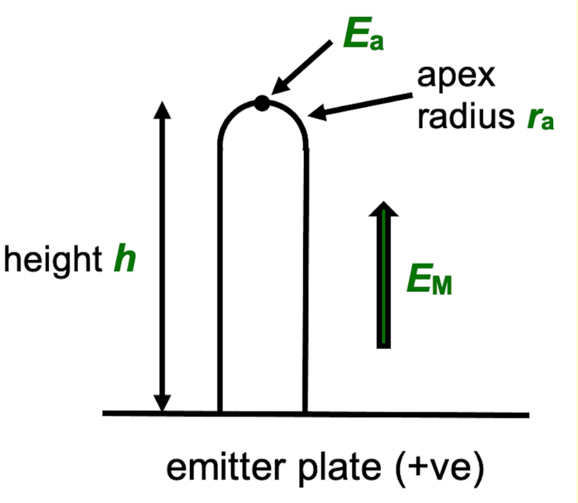

For such posts, an important physical parameter Is the apex sharpness ratio (ASR) σ_a defined by

$$\sigma_a \equiv h/r_a ,$$

where r_a is the apex radius of curvature.

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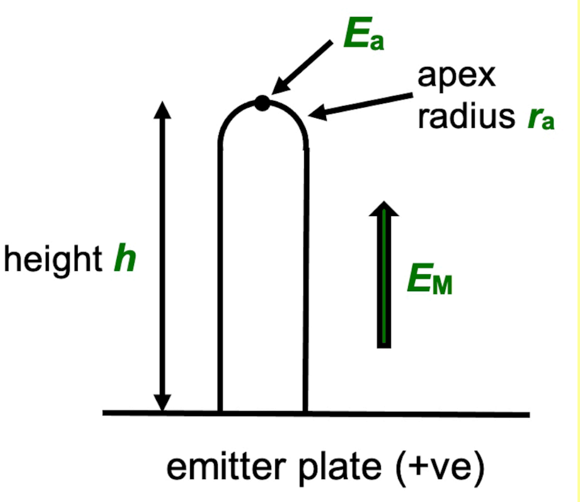

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$$\sigma_a \equiv h/r_a \,,$$

where r_a is the apex radius of curvature.

For such posts, it is now thought that good formulae for the apex FEF can (often or always) be written in the form:

$$\gamma_a = c_a(\sigma_a) \cdot \sigma_a \,,$$

where the apex sharpness coefficient $c_a(\sigma_a)$ is shape-dependent.

Hemi-prolate-spheroidal (HPS) model

Recent work by de Assis, Dall'Agnol and Forbes has generated improved expressions for $c_a(\sigma_a)$ for two important post shapes.

For the hemi-prolate-spheroid (HPS) post model, an exact formula is:

$$c_a(\sigma_a) = (\tau_a/\sigma_a)^{3/2} / \left[\ln\left\{\tau_a^{1/2} + \sigma_a^{1/2}\right\} - (\tau_a/\sigma_a)^{1/2} \right],$$

where

$$\tau_a \equiv \sigma_a - 1 .$$

For tall/slender HPS posts (say σ_a>40), this reduces to the simpler form

$$c_a(\sigma_a) \approx 1/[½\ln\{4\sigma_a\} - 1] .$$

Hemisphere-on-cylindrical post (HCP) model

For the hemisphere-on-cylindrical-post (HCP) model, a good approximate formula (with error ε of magnitude 0.15 % or less, over $1 \leq \sigma_a \leq 1000$) is:

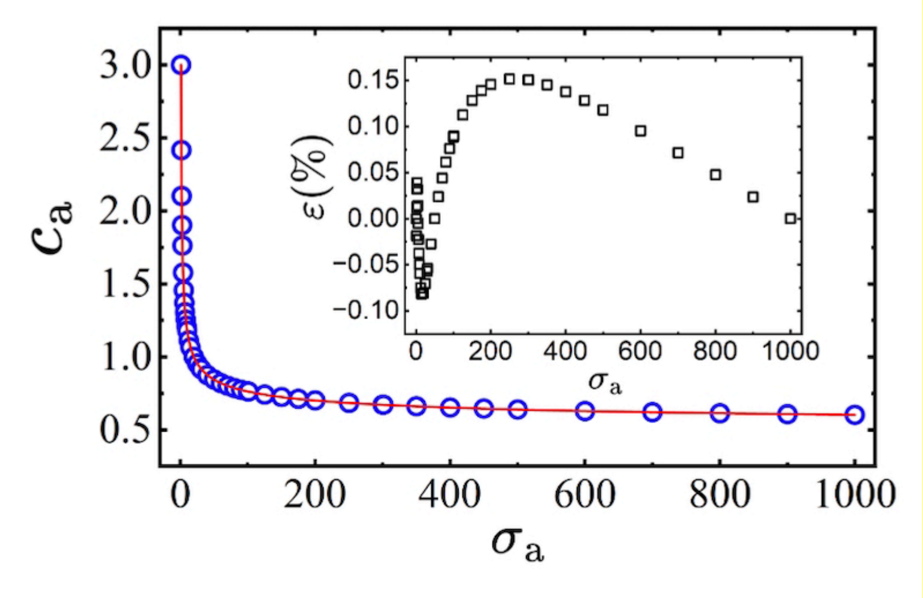
$$c_{\mathrm{a}}(\sigma_{\mathrm{a}}) = c_{\mathrm{L}} + \frac{3 - c_{\mathrm{L}}}{1 + \alpha \ln\left(\frac{1+\beta\sigma_{\mathrm{a}}}{1+\beta}\right)}$$

where: $\alpha = 1.099(1)$; $\beta = 1.255(6)$; $c_L = 0.2573(3)$. [The figure in brackets is the uncertainty in the last significant figure.]

As shown in the figure alongside, $c_a(\sigma_a)$ decreases monotonically from $c_a(1) = 3$, as σ_a increases.

The decrease is initially rapid, but then flattens off.

[The formula is derived by fitting to high-accuracy finite-element numerical simulations.]



D

Problems with FE literature

Literature weaknesses

There are many misunderstandings/weaknesses in FE technological literature. "Weak practices" in published papers include the following.

Significant problems

1. **Statement/use of the elementary FE equation, rather than some form of Murphy-Good low-temperature FE theory (which predicts much higher local emission current densities).**
2. **Failure to distinguish between local and "emitter-average" current densities.**
3. **Failure to identify "non-ideal" FE systems, causing spurious results for extracted field enhancement factors.**
4. **Citing the 1928 Fowler-Nordheim paper as the only theoretical reference.**
5. **Confusion caused by using the name "Fowler-Nordheim equation" for four different equations that make different theoretical predictions.**

Literature weaknesses

Minor problems

6. Discrepancies associated with the use of around 20 different theoretical approximations for the mathematical factor v_F in Murphy-Good theory. [Note: (a) most of these approximations are now obsolete; (b) some of the popular ones make the unhelpful false prediction that a Fowler-Nordheim plot will be exactly straight.]
7. Use of Gaussian system equations and/or quantities rather than the modern system of equations and quantities (the "ISQ") that forms the basis for SI units.
8. Use of dimensionally inconsistent equations.
9. Use of incorrectly rounded numerical values for the Fowler-Nordheim universal constants.
10. Inconsistent use of the terms "Fermi energy" and "chemical potential".

The problem with the elementary FE equation

The elementary FE equation significantly underpredicts local emission current densities, but is still often stated. This is particularly unhelpful in the following contexts.

(a) When developing new forms of large-area electron source, there is no commercial or medical merit in writing down an equation that seriously underpredicts their performance.

(b) When discussing the electrical-breakdown behaviour of large particle accelerators, (such as those at CERN, and similar) there is no merit in using an equation that seriously underpredicts expected emission currents.

(c) When military research is using Murphy-Good theory in the context of modelling FE devices for military applications, it is unhelpful for a stream of published papers to apparently undermine that research by implying (falsely) that the theory being used is incorrect.

The problem with the elementary FE equation

It seems that the peer review system in FE often doesn't work well:

***Quite often, neither authors, nor reviewers, nor editors can detect that an unhelpful out-of-date equation is being used.**

***Writing "Comments" on specific papers appears to be of limited effectiveness. In any case there too many "theoretically weak papers" for this to be practicable. (One arrives at my computer about every two weeks.)**

***Writing textbooks and review articles seems ineffective: there are already at least 28 of them (all of which recommend use of MG low-temperature FE theory – or something more sophisticated).**

Two proposals that might help deal with this problem are:

(1) Writing individually to relevant corresponding authors, with a copy of a recently written (and now published) summary of basic FE theory. [Find by websearch on title: "Summary overview of present state of basic field electron emission theory"].

(2) Encouraging people to consult the Google AI assistant ....

What Google thinks (1)

What is the best modern theory of field (electron) emission?

AI Mode | All | Images | Short videos | Videos | Shopping | Forums | More | Tools

AI Overview

The best working framework for modern field electron emission is **Extended Murphy–Good (EMG) theory**, which builds upon the foundational Murphy-Good (MG) 1956 theory to correct errors in the original 1928 Fowler–Nordheim (FN) equation. AIP Publishing +3

Why Murphy–Good Outperforms Original Fowler–Nordheim

- **Barrier Shape Correction:** The original FN theory assumed an unrealistically sharp triangular barrier. Modern MG theory incorporates a Schottky-Nordheim (SN) barrier that properly accounts for image-charge potentials and rounded barrier tops. royalsocietypublishing.... +2
- **Accuracy:** The elementary FN equation drastically underpredicts true emission current densities (often by a factor greater than 100) and should be avoided in rigorous research. AIP Publishing
- **Data Interpretation:** Traditional linear Fowler–Nordheim plots hide minor natural curvatures caused by proper barrier physics. Modern methodology prefers **Murphy–Good plots** to accurately extract physical parameters like the local work function and emission area. royalsocietypublishing.... +1

What Google thinks (2)

Frontiers in Modern Emission Theory

- **Curved and Nanometric Emitters:** Contemporary research generalizes planar models to handle ultra-sharp nanometer-radius tips (such as nanotips and individual carbon nanotubes) where geometric asymmetry and apex-shape scaling heavily influence quantum tunneling. Nature +1
- **Thermal-Field Transitions:** Advanced models (such as the Jensen-style General Thermal-Field framework) bridge cold field emission and high-temperature thermionic emission for intermediate operational regimes. AIP Publishing

If you are working on a specific application, let me know:

- Are you analyzing **experimental I-V data** (like Fowler-Nordheim or Murphy-Good plots)?
- Are you modeling **nanoscale emitters** or broad-area electrodes?

I can provide more tailored equations or plotting protocols.

Part E: Some future needs

Applications – large-area sources

Large-area-source applications (RGF's assessment)

(a) Established

- **Some types of computer flash memory**
- **X-ray generators for medical applications**
- **Ionization devices for miniature mass spectrometers**
- **Theoretical models for electrostatic simulations**

(b) Promising

- **X-rays generators for other tasks**
- **Sensors**
- **Radiation-hard air-channel or vacuum-channel transistors**

(c) Maybe

- **Microwave generators and similar devices**
- **Spacecraft neutralization applications**
  (especially in connection with electric propulsion)
- **Tether-type spacecraft propulsion devices**

cont./

Most immediate "theoretical" research tasks

For low-temperature FE, I think the most immediate research tasks are:

***Terminology clarification/unification/improvement.**

***Getting obvious (to FE theoreticians) error out of the literature.**

***Clearer statement of Murphy-Good low-temperature FE theory, including its derivation and ALL its limitations.**

***Low temperature FE theory for sharply curved emitters, including: (a) theory for current prediction and for range of applicability; (b) data validity tests; and (c)data analysis procedures.**

***Getting atoms into metal FE theory, initially for planar emitters.**

***Continuing to improve our procedures for making reliable comparisons between FE theory and experiment.**

***Background improvements in underlying science.**

***Developing a consensus-based roadmap for detailed further development of FE theory.**

Some of these things are already in progress, or are now well advanced, and need to be continued.

Thanks for your attention

FE theory "tasks"

Mainstream tasks (as defined earlier):

Task M0: **Terminology clarification/improvement**

Task M1: **Emission theory based on SPME methodology**

Task M2: **Theory for sharply curved emitters**

Task M3: **Getting atoms into metal FE theory, initially for planar emitters**

Task M4: **Investigating coherence issues**

Task M5: **Investigating fundamental quantum mechanical issues**

Task M6: **Understanding picoscale FE**

Terminology: "electronically ideal"

The term FE (experimental) system covers all aspects of an experimental apparatus that can affect measurements: emitter characteristics; apparatus arrangements of all kinds; and all aspects of the experimental measurement arrangements.

An electronically ideal FE system is one where current-voltage relations are determined by emission physics and emitter electrostatics alone, and there are NO "system complications" (e.g., series resistance, space-charge, voltage loss along emitter).

With an electronically ideal system, the local surface field (ESPG) at any specific point on the emitter surface is proportional to the measured voltage, with the local proportionality constant independent of voltage.

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Conventional data-analysis procedures, such as Fowler-Nordheim plots, are strictly valid only for electronically ideal FE systems. If applied to non-ideal FE systems, conventional procedures may generate spurious results.

Terminology: SPME methodology

Nearly all FE technological applications model the emitter surface by the Sommerfeld 1927 free-electron model.

This model:

(a) takes the emitter surface to be smooth, planar, and of large lateral extent;

(b) disregards the atomic structure of the real emitter, and the possible theoretical role of atomic-level wave-functions.

I call the use of the Sommerfeld model in FE theory "smooth planar metal emitter (SPME) methodology".

2: Identification of scientific tasks

FE emission physics "tasks"

Mainstream tasks:

Task M0: Terminology clarification/improvement

Task M1: Emission theory based on SPME methodology

Task M2: Theory for sharply curved emitters

Task M3a: Getting atoms and band-structure effects into metal FE theory, initially for planar emitters

Task M4: Investigating coherence issues

Task M5: Investigating fundamental quantum mechanical issues

Task M6: Understanding picoscale FE

Current/voltage "tasks"

Associated "current-voltage-related" tasks:

Task A1: **Establishing how to measure local surface field (ESPG) reliably**

Task A2: **Establishing how to make reliable comparisons between theory and experiment.**

Task A3: **Implementing temperature effects in emission theory**

Task A4: **FE systems engineering, especially for non-ideal systems**

Task A5: **Semiconductor and "effective mass" emission effects**

Task A6: **Effects associated with carbon nanotubes, graphene, and other carbon-based materials**

Task A7: **Theory for "oscillating radio-frequency ES fields"**

Task A8: **Theory for laser-created applied electromagnetic (EM) fields**

FE theory "tasks"

Tasks related to
field emitter electrostatics (ES) and field-induced atomic movement

Task E1: Overview of existing analytical and numerical approaches

Task E2: Electrostatics: array effects

Task E3: ES: proximity and "screening" effects (single-tip and array)

Task E4: Basic physics of electroformation (metals)

Task E5: Basic physics of electroformation (carbon nanostructures)

Task E6: Exploration of possible electrohydrodynamic (EHD) effects (e.g., "EHD explosion")

Tasks related to research-literature integrity

Task R1: Problems with basic definitions

Task R2: Unhelpful use of term "Fowler-Nordheim equation"

Task R3: Entrenched misunderstandings/errors in many published papers

Theoretical levels in Task M1

Within Task M1 (use of SPME methodology), I currently define four theoretical levels.

Level 0: **Is a modern (widely used) simplified version of Level 1, based on the so-called elementary FE equation.**

Level 1: **Is based on the original 1928 Fowler-Nordheim (FN) FE equation, which uses an exactly triangular (ET) tunnelling barrier.**

Level 2: **Is based on the (zero-temperature version of) the 1956 Murphy-Good (MG) FE equation, which uses the Schottky-Nordheim (SN) tunnelling barrier. Uses Nordheim parameter, *y*.**

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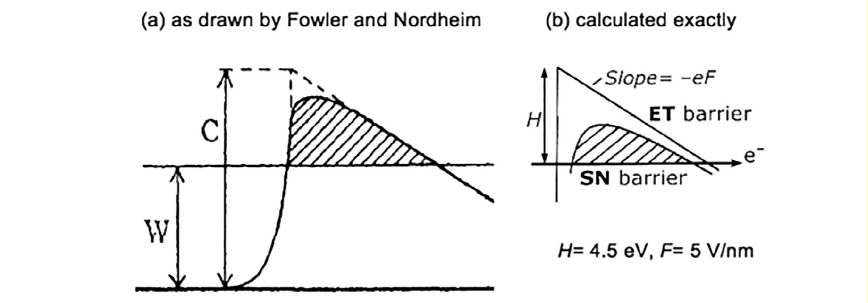
Level 3: **"21st Century planar FE theory". Is based on so-called Extended MG (EMG) FE equation. This is "experiment-oriented" rather than "prediction oriented". Uses Gauss variable x [=y^2] and scaled field f, instead of Nordheim parameter. Uses special mathematical function (SMF) $v_{FD}(x)$ identified by Forbes and Deane, together with related SMFs. Distinguishes between mathematics and modelling.**

Theoretical levels in mainstream Task M1

Levels 0 and 1 are based on a mathematical mistake made by Fowler and Nordheim in 1928, and a subsequent modelling misjudgment. For data interpretation, these levels need to be abandoned.



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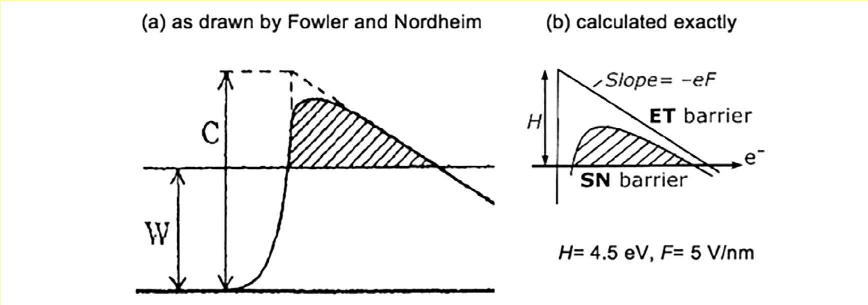


Level 2 involves significantly better physics than levels 0 and 1.

Level 3 involves the same physics as level 2, but involves significantly better mathematics and improved scientific procedure.

Level 0 theory can be useful in preliminary theoretical studies of complex situations, but will not yield numerically accurate results.

Data-analysis methodology at Level 3

At level 3 there is a "current-voltage analysis methodology for electronically ideal systems". [Call this a "ideal-system 3-part analysis methodology".] This methodology involves:

1) **Predictive formulae for current-voltage characteristics.**
2) **Validity checks for whether system is electronically ideal, namely**
   - **(a) Date-plot linearity, and/or**
   - **(b) The "orthodoxy test", and/or**
   - **(c) The "magic emitter" (or "implausible field values") test.**
3) **Data-analysis theory, for electronically ideal systems, for Fowler-Nordheim plots and for Murphy-Good plots.**

There is a case for arguing that all technological FE current-voltage data-analysis should now use level 3 theory (at least for the time being).

3: Deeper discussion of selected tasks

Current and future theoretical tasks

Current and future mainstream theoretical tasks can be classified into:

(a) Immediate tasks.

(b) Near-future tasks.

(c) Longer-term tasks.

Current and future theoretical tasks

Current and future mainstream theoretical tasks can be classified into:

- **(a) Immediate tasks.**
- **(b) Near-future tasks.**
- **(c) Longer-term tasks.**

In my view, immediate tasks relate to

- **R2: Unhelpful use of term "Fowler-Nordheim equation"**
- **R3: Entrenched misunderstandings/errors in many published papers**

3.1: Immediate tasks

Equation nomenclature

In my approach, each of the levels has its own named equation, as follows.

Level 0: **Elementary FE equation:**

$$J_L^{el} \equiv a\phi^{-1} F_L^{2} \exp[-b\phi^{3/2}/F_L]$$

Level 1: **1928/29 Fowler-Nordheim (FN) FE equation:**

$$J_L^{FN} \equiv P^{FN} a\phi^{-1} F_L^{2} \exp[-b\phi^{3/2}/F_L]$$

Level 2: **1956 Murphy-Good (MG) FE equation (zero K form):**

$$J_L^{MG0} \equiv \mathrm{t}_F^{-2} a\phi^{-1} F_L^{2} \exp[-\mathrm{v}_F b\phi^{3/2}/F_L]$$

Level 3: **Extended MG (EMG) FE equation:**

$$I^{EMG} \equiv A_{fC}^{SN} a\phi^{-1} F_C^{2} \exp[-\mathrm{v}_F b\phi^{3/2}/F_C]$$

In the literature, the first three equations are all called "the Fowler-Nordheim equation". This seems to have generated much confusion. It is strongly recommended that distinguishing names be used. In particular, the level-2 equation should NOT be called "the Fowler-Nordheim equation".

Literature misunderstandings

R3: An urgent task is to deal with misunderstandings in FE technological literature. Common errors that appears in many published papers are:

Significant errors

1. **Failure to distinguish between local and macroscopic current densities.**
2. **Use of level 0 theory.**
3. **Failure to identify non-ideal FE systems, causing spurious results for field enhancement factors.**
4. **Very large undetected discrepancies between theory and experiment.**
5. **Citing the 1928 Fowler-Nordheim paper as the only theoretical reference.**

Minor problems

6. **Use of Gaussian system equations rather than the modern system of equations (the "ISQ") that forms the basis for SI units.**
7. **Use of dimensionally inconsistent equations.**
8. **Use of incorrectly rounded values for the Fowler-Nordheim universal constants.**

Literature misunderstandings

It is clear that, in the context of technological FE, the peer review system has partially failed, and that a situation exists that sociologists call "pathological science".

It is unclear what is the best method of dealing with this.

It seems possible that this situation could become evidence examined by a Government or parliamentary enquiry into the integrity of scientific publishing.

3.2: Near-future tasks

Near-future tasks

Background tasks

M1: Emission theory based on SPME methodology
[In particular, encouraging use of level 3 FE theory]

A4: FE systems engineering, especially for non-ideal systems
[Many detailed problems]

R1: Problems with basic definitions
[See next slide]

M0: Terminology clarification/improvement
[e.g., use "electrostatic (ES) field" instead of "electric field"]

Research development tasks

A7: FE Theory for "radio-frequency oscillating ES fields"
[Create "Ideal-system 3-part analysis methodology" for this case]

M2: Theory for sharply curved emitters

M3: Getting atoms into metal FE theory, initially for planar emitters

R1: Problems with basic definitions

R1: Problems with basic definitions

There seem to be problems with the formal definitions of the following basic quantities:

(a) Fermi level and Fermi energy
(see Poster at https://doi.org/10.13140/RG.2.2.32249.58724)

(a) Voltage

(b) Chemical potential

There also seems a need to introduce a fundamental constant n_1 that represents the "amount of substance of a system that contains one specified entity".

FE theory "tasks"

Mainstream tasks (as defined earlier):

Task M0: **Terminology clarification/improvement**

Task M1: **Emission theory based on SPME methodology**

Task M2: **Theory for sharply curved emitters**

Task M3: **Getting atoms into metal FE theory, initially for planar emitters**

Task M4: **Investigating coherence issues**

Task M5: **Investigating fundamental quantum mechanical issues**

Task M6: **Understanding picoscale FE**

M2: Theory for sharply curved emitters

Obvious initial problem is theory for Earthed spherical emitters. Other shapes/situations can follow later, where needed.

In every case an "ideal-system 3-part analysis methodology" (or equivalent) is needed. This may differ in detail from case to case.

As a result of past work by Xanthakis, Kyritsakis and colleagues, much of the necessary theory for Earthed spherical emitters is in place.

A "bridging step " is that experimentalists will need to get familiar with numerical multilinear regression.

Initially, it might be useful to get familiar with using regression techniques in the context of SPME methodology [some authors already use regression approaches].

M3: Getting atoms into metal FE theory

FE needs to become completely numerical, with algebraic formulae fitted to numerical results where appropriate.

Bridging steps will/may probably include:

1. **Review of existing methods for numerical solution of 1-D Schrödinger equation. [Cahay, Jensen and Mayer have what appear to be slightly different methods, and there may be others.]**
2. **Systematic comparisons between numerical and analytic treatments of the Schottky-Nordheim barrier (including the metal potential-energy). [This is a cross-validation exercise, to which Mayer and others have already made a significant contribution.]**
3. **In the above potential-energy variation, review of "travelling-wave resonance effects" between the barrier and the emitter surface. [Various theoretical treatments exist.]**

The atomically flat metal surface

An "exact" "atomic-level" theory of FE from a flat metal surface will probably involve solving the following problems.

1. **How to self-consistently use density functional theory (DFT) to calculate the potential energy (PE) structure in which the escaping electron moves, taking into account the exchange-and-correlation interaction between this electron and the emitter surface.**
2. **How to chose "paths of integration" in this PE structure.**
3. **How, for an electron with "given energy characteristics", to sum over the different paths of integration associated with its motion.**
4. **How to sum over the distribution of permitted electron energies, taking into account the statistical mechanics of the situation, to obtain a value for emission current density.**
5. **How to fit numerics to the results, to yields an algebraic expression for current density in terms of surface work function, applied field (ESPG) and temperature.**

Some relevant work already exists, but there are also conceptual problems.

3.3: Longer-term tasks

How to describe a tunnelling electron

Some fundamental issues are:

1. **What does the squared modulus of the wave-function represent: "the result of a measurement" or "the distribution of electron matter?"**
2. **If the former, what does it mean to find an electron at a place where it has negative kinetic energy [in accordance with widely published populist "interpretations" of quantum mechanics]? [Or are the populist interpretations "bad physics" ? (Yes, in my view).]**
3. **Could a "real electron" have negative kinetic energy? [No, in my view.]**
4. **Hence, the wave-function for a real electron must be distributed and must always "stick out at one or both ends of the barrier."**
5. **This suggests to me that there may be TWO types of wave-function: one associated with actual matter distribution, the other with "pathway choice" (in FE, associated with transmission probability).**
6. **Incidentally, what holds a distributed electron together?**

How to describe a tunnelling electron

Some further questions are:

7. **How long is an electron ?**
8. **If an electron is 20 nm long, then—in the uncertainty principle—what do we mean by the "position of an electron" ?**
9. **Does the concept of "wave-particle duality" need to be replaced by something more sophisticated and more carefully formulated. [What do we mean by a "particle" ?]**

How to calculate tunnelling rate-constant

The main issues here are as follows.

1. **Landau and Lifschitz (LL) assume that the JWKB approximation is only valid in cartesian coordinates. This may cause complications for all non-planar tunnelling theory, including FE theory in Task M2.**

2. **LL seem to have used a "model of reality" / "interpretation of quantum mechanics" that is different from the conventional ones. You might call it the "many simultaneous paths in a single world" interpretation. They split the electron into many infinitesimally small elements, each of which follows a different path and has a different tunneling probability. They then do a sum when all the elements have emerged from the barriers.**

   The following issues arise

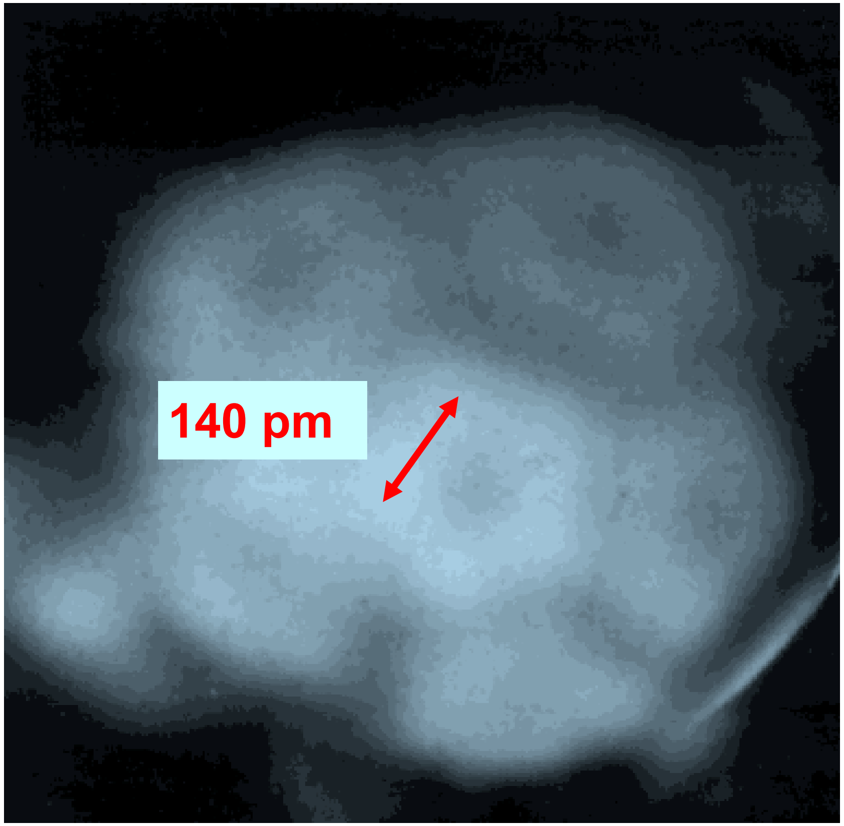
   (i) How to decide the actual paths?

   (ii) Is summing probabilities (rather than amplitudes) correct ?

   (iii) How to deal with image effects when the atom is close to a surface?

3. **The function $v_{FD}(x)$ introduced by Forbes and Deane can be applied to field ionization and provides better mathematics than the LL treatment.**

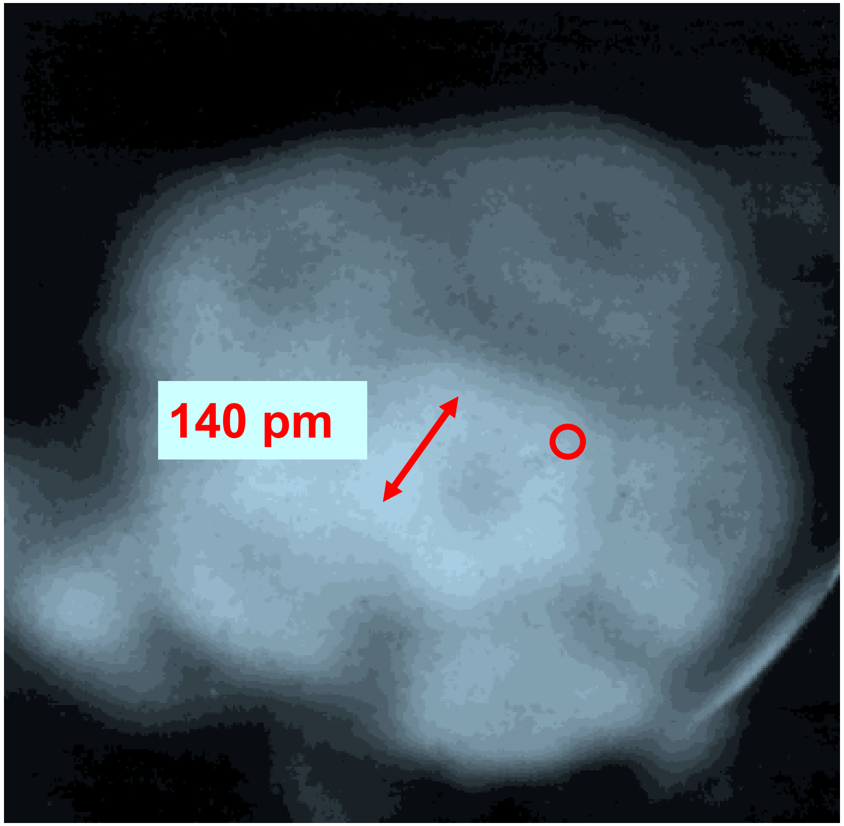
M6: Understanding picoscale FE



Field electron microscope image of end of carbon nanotube, showing five-sided carbon-bond ring, with inter-nuclear distance about 140 pm.

[Courtesy: Y.Saito, K.Hata, T.Murata, Jpn J. Appl. Phys 39 (2000) L271–L272, Fig. 2(a).]

M6: Understanding picoscale FE



30 pm diameter probe-hole

Field electron microscope image of end of carbon nanotube, showing five-sided carbon-bond ring, with inter-nuclear distance about 140 pm. [Courtesy: Hata]

With outline of 30-pm probe-hole superimposed.

Conclusion

There is very much work to be done.

Estimated timescale: 20 to 100 years

Thanks for your attention

Maths & Physics
Quantum Foundations & Technologies Group

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Extended abstract (as accepted), p.1

Exploring the Future of Field Electron Emission Theory

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***Abstract*—This presentation relates to the theory of field electron emission (FE), with focus on the theory of current-voltage characteristics and special emphasis on the theory of electron transmission probability. After a survey of where we have got to after about 100 years of investigation, the intention is to present a high-level roadmap for what (in the author's view) needs to be done in order to put the basic principles of FE theory onto a more sophisticated and complete scientific basis. Possible difficulties will be identified.**

***Keywords*—Field electron emission (FE) theory, FE Systems Engineering, FE current-voltage theory, transmission probability.**

I. General Introduction

The oral presentation summarised by this extended abstract is a new attempt, in 2024, to offer a high-level roadmap for the future development of "mainstream" field electron emission (FE) theory. By "mainstream" is meant theory relating to FE from metal elements in the "deep tunneling" regime (also known as the "cold FE" regime), including theory relating to measured FE current-voltage and energy-distribution characteristics, and their interpretation.

There are many important/specialized aspects of FE theory outside this mainstream, including temperature-induced and photon-induced effects, FE from materials that are not metal elements, and FE from rough surfaces. Research in these areas is welcome, but we shall not get non-mainstream theory exactly correct if mainstream theory is not exactly correct.

The general structure of the presentation and abstract is as follows. A scientific and technological introduction reviews: (a) the relevance of FE; (b) the current state of "mainstream" FE transmission probability theory; and (c) issues relating to current-voltage measurement. Attention then focuses on the long-term issues, especially transmission probability theory.

To save space, references are given to earlier papers by the author. This presentation is not intended as a review of all relevant work. A copy of the oral presentation slides will be mounted on ResearchGate, and are most easily be found by a websearch on the title. This copy will contain a more comprehensive list of references.

II. Scientific and Technological Background

A. The relevance of field electron emission (FE)

• Many technologies are based on FE. FE theory can assist interpretation of experimental results, and/or guide R&D.

• FE theory is important in electrical breakdown theory.

• FE is one of the paradigm examples of quantum tunnelling. As such, future developments in FE theory might have implications elsewhere in science, for example in quantum biology. (DNA mutation involves thermally activated tunnelling of hydrogen atoms or ions [1].)

B. FE transmission probability theory

FE literature contains very many variants of transmission probability theory, but the commonest variants assume the Sommerfeld model, take the emitting surface as planar and of large lateral extent, and disregard the role of atomic-level wave functions. This has been called *smooth planar metal-emitter (SPME) methodology* [2]. Within this methodology, four often used "levels" of FE theory have been defined [2].

- ***Level 1 – FN FE theory.*** Theory given by Fowler & Nordheim in 1928 and corrected by Stern et al. in 1929 [3].
- ***Level 0 – elementary FE theory***. A modern simplified version of Level 1 theory that omits a pre-exponential factor. Level-0 theory is widely used by experimentalists (albeit undesirably), and is also used in undergraduate teaching.
- ***Level 2 – MG FE theory***. Theory given by Murphy & Good in 1956 [4], but best presented using modern SI-type equations. MG FE theory uses an exponent correction factor "v" related to image effects, and makes predictions typically around 100 times larger than Level-1 theory. "v" was originally written in terms of the *Nordheim parameter* "*y*".
- ***Level 3 – Extended MG (EMG) FE theory***. A variant of MG FE theory, based on modern understanding of the mathematics of "v", developed from 2006 onwards. "v" is recognized as a special solution $v_{FD}(x)$ of the Gauss Hypergeometric Differential Equation, using the *Gauss variable x*. In FE modelling, *x* is set equal to the *scaled field* f [$=y^2$]. EMG theory is also "experiment facing" in its definition of area. EMG FE theory is described in [5,6].

C. Issues related to current-voltage measurement

In principle, the prediction and interpretation of FE current-voltage measurements is part of a specialized (but under-developed) branch of electronic/electrical engineering recently called [7] *FE Systems Engineering*. Traditional current-prediction and data-analysis methodologies assume that FE systems are *electronically ideal* (i.e., that a characteristic local electrostatic-field magnitude F_C related to the emission process is directly proportional to the measured voltage V_m, with the *local voltage conversion length (LVCL)* $\zeta_C = V_m/F_C$ *constant* (independent of V_m). With modern emitters this is often not the case [7]; thus values of extracted emitter characterization parameters may be spurious.

For SPME-methodology data-analyses, validity tests [7,8] can now check whether systems are electronically ideal, and should now be part of normal research practice.

Other serious difficulties with current technological data-analysis practice, including the widespread use of multiply defective equations, are described in [7–9].

Extended abstract (as accepted), p.2

III. General needs for future progress

• There is inconsistency in the terminology used in FE literature. "Debabelization" and standardization are needed.

• A background difficulty is that some definitions in International Standards are not "fit for purpose for FE", in particular: "Fermi level", "voltage", and "chemical potential".

• FE literature has about 20 different approximations that can be construed as approximations for $v_{FD}(x)$, with some making the false prediction that a FN plot should be linear. "Good" and "high-precision" (error better than 10^{-9}) approximations for $v_{FD}(x)$ are now known [6]. Many older approximations are thus now obsolete, and their use should be discontinued.

• For the present, it would be helpful (particularly in making comparisons) if all current-voltage data-analysis were carried out using a single preferred methodology based on Level-3 theory. This would be "better R&D practice" than commonly used methods, and a good starting place for urgently needed further development of FE theory and data-analysis. Using MG plots [5], rather than FN plots, might be part of it.

• There is a need for more-careful explicit analysis of the experimental conditions and apparatus needed for reliable comparison of FE theory and experiment (see [10,11]). In the last 100 years there have been tens of thousands of experiments on the electrostatics of field emitters, but only one reliable experiment on the basic theory [12]. This generated a result intermediate between Level 1 theory and Level 2 theory, but closer to Level 2 theory. This result needs to be confirmed for metal surfaces other than tungsten (110).

• There are at least 12 reasons [7] why modern FE systems might not be electronically ideal. Some, like space-charge effects, have well developed theory. Other aspects of FE Systems Engineering require significant further development.

IV. Some immediate needs

For emission theory and related data-analysis procedures, two lines of development need to be progressed.

A. Theory for smooth-surface curved emitters

SPME methodology breaks down for emitters that are sufficiently sharply curved, say with apex radius of curvature less than about 20–50 nm. The first obvious advance is to develop analysis methodology (transmission theory, specific statistical mechanics, emission current theory, validity checks, and data-analysis procedures) for electronically ideal FE systems based on "Earthed" spherical emitters. Much progress has been made by Kyritsakis [13]. Theory for more complicated shapes (e.g., [14]) may be needed later.

Data analysis for such systems needs to use numerical multilinear regression techniques. A useful step would be to apply such techniques in SPME methodology and compare extracted parameters with those from FN and MG plots.

B. Getting atoms into planar-surface metal FE theory

An urgently needed step forwards is to "get the atoms into planar-surface metal-element FE theory." Although there have been earlier quantum-mechanical treatments, recent calculations of potential-energy (PE) structures use modern density functional theory (e.g. [15–16]).

It is clear that the future of mainstream FE theory will be "completely numerical". For examples, see [15–17].

An useful first step would be to review comparisons between conventional treatments of MG theory and existing fully numerical treatments, such as that of Mayer [17].

A later need is to explore how to choose "paths of integration" (or a "typical path") through a calculated PE structure. A partial guide to this may be to look again at the theory of field ionization of a hydrogenic ion, as derived from the work of Landau and Lifschitz (LL) [18]. But, here, more fundamental quantum mechanical problems seem to intrude.

V. Fundamental quantum-mechanical problems

Briefly, it not obvious that existing quantum mechanics is "fully fit for purpose" for all FE theoretical problems. Limited space allows only an indication here. The oral presentation provides longer discussion of the following identified issues.

• The "real" nature of an electron and apparent inadequacy of the Copenhagen interpretation of quantum mechanics. ("What is the length of a tunnelling electron?" and "Can a real electron have negative kinetic energy": [No, in my view.])

• The unduly simplistic nature of "wave-particle duality" and "the uncertainty principle", in the context of FE theory.

• The apparent need to use two *different* interpretations of the wave-function in tunnelling problems involving correlation-and-exchange effects (one to describe "real behavior", the other to discuss the behavior of wave-packets that can split).

• The apparent use by LL of a specialized model of reality in their tunnelling theory. One might call this the "many simultaneous paths" model, in which an electron is considered split into infinitely many infinitesimal elements, each of which follows a different path through the barrier and would have different transmission characteristics. Several issues arise here, relating to coherence and to actual paths.

• Whether final FE emission is incoherent or coherent.

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